Study of blends of styrene–butadiene rubber and poly(vinyl chloride): 1. Analysis of bending in the elastic range

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The bending behaviour of styrene-butadiene rubber/poly(vinyl chloride) (SBR/PVC) blends has been studied. By solving differential equations and applying an elastic mechanics calculation, we propose a method of determining the deflection of SBR/PVC blends in their elastic range, comparing the calculated results with the experimental data. Factors which influence the deflection of SBR/PVC blends are discussed.

(Keywords: styrene–butadiene rubber; poly(vinyl chloride); bending behaviour; elastic range)

INTRODUCTION

A search for increasing the impact strength of poly(vinyl chloride) (PVC) is currently under way. Blending is most commonly used to improve the mechanical properties of polymeric materials[1]. In the literature much attention has been paid to blends of PVC with nitrile rubber (NBR), chlorinated polyethylene (CPE) and poly(ethylene-co-vinylacetate) (EVA)[2,3]. Using styrene-butadiene rubber (SBR) as toughening agent, we have also investigated the improvement in impact resistance of PVC. Earlier work in our laboratory showed that the blending of SBR into PVC resulted in a marked improvement in the impact strength of the PVC. When the weight ratio of SBR/PVC is 7/93, the Charpy impact strength of the blend was 18–20 kg cm cm^{-1} at 25°C[4].

In an attempt to broaden the applied field of this new SBR–PVC blend, we have investigated its mechanical properties, rheological characteristics and morphological structure.

In this paper we will study the bending deformation of the SBR/PVC blend acting as a simple supported beam with the load in the range of elasticity; deriving prediction equations for the deflection. It is shown that the data for the SBR/PVC blends fit the predictions well. Thus we provide a theoretical method for estimating the bending deflection of these materials. The importance of the factors that influence the deflection are also discussed.

THEORY

The deflection curve equation in case of large deformation

A segment of an initially straight beam is shown in a deformed state in *Figure 1*. The deflected axis of the beam, i.e. its elastic curve, is shown bent into a radius ρ. The vertical displacement of all points located on the axis is called the deflection, y, and the angle, θ, made by the tangent line with the original axis line slope. If we are considering a simple supported beam, the bending deformation under a concentrated load may be calculated. If our attention is confined to the study of small deformations and further, if the behaviour of the material is considered only in its elastic range, the equation takes the form[5]

$$\left.\begin{aligned} y_{1/2} &= \frac{1}{48}\cdot\frac{Pl^3}{EI} \\ I &= \frac{1}{12}bd^3 \end{aligned}\right\} \tag{1}$$

where $y_{1/2}$ is the deflection at the middle point of span, P is load, l is the span, E is Young's modulus, I is the moment of inertia of area, b is width of the test specimen and d is the thickness of test specimen. In general, the higher the toughness of the material is, the larger is the deflection. Perhaps equation (1) may be considered unsuitable for the SBR/PVC blend due to its high toughness and it is necessary, therefore, to derive some other calculating equations.

Here we will only consider the left part of the beam, owing to its symmetry. In Cartesian coordinates the differential equation of the deflection curve given above is

$$\left.\begin{aligned} \frac{\mathrm{d}^2y/\mathrm{d}x^2}{[1+(\mathrm{d}y/\mathrm{d}x)^2]^{3/2}} &= ax \\ a &= \frac{P}{2EI} \\ I &= \frac{1}{12}bd^3 \end{aligned}\right\} \tag{2}$$

where x is the axial coordinate and the other symbols are as above. In general, the solution of such an equation is very difficult to achieve[6] and, therefore, the solution has not yet been arrived at. We have solved this equation by means of a series expansion, and reported the result as follows:

If we make $\mathrm{d}y/\mathrm{d}x = t$ or $\tan\theta = t$, we have

$$\cos\theta\,\mathrm{d}\theta = ax\,\mathrm{d}x \tag{3}$$

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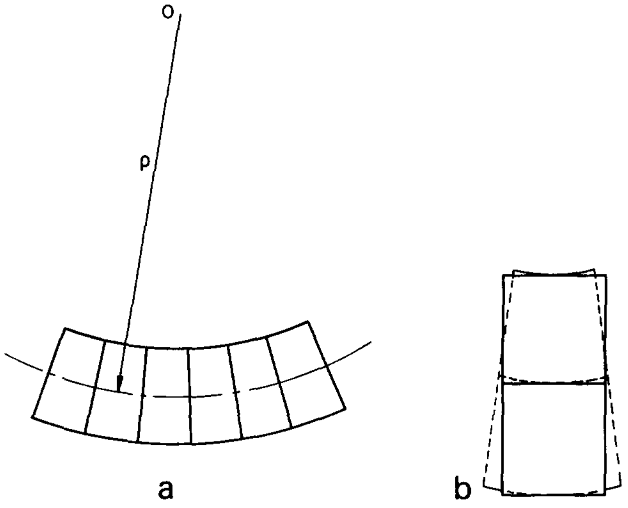

Figure 1 Deformation of a beam segment on bending

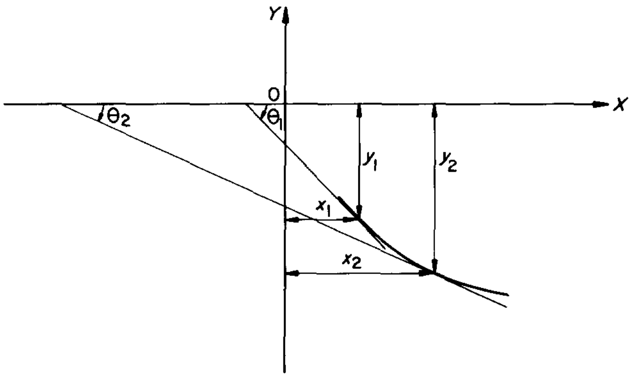

Figure 2 Diagram of deflection y and slope θ

Integrating equation (3) then

$$\sin\theta = \tfrac{1}{2}ax^2 + C \tag{4}$$

By a trigonometric equivalent expression, we have

$$\sin\theta = \frac{t}{(1+t^2)^{1/2}}$$

and by substituting it into equation (4), we obtain

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{(C+\frac{1}{2}ax^2)^2}{1-(C+\frac{1}{2}ax^2)^2}\right)^{1/2} \tag{5}$$

(the minus indicates that on the left of the beam $t=\tan\theta<0$). Boundary conditions:

$$y(0)=0 \quad \text{and} \quad y'\left(\frac{l}{2}\right)=0$$

Then we obtain the value of integration constant:

$$C = -\tfrac{1}{8}al^2$$

Putting $x=\frac{1}{2}l\cos\omega\mathrm{d}\omega$; then

$$y = -\tfrac{1}{16}al^3\int\frac{\cos^3\omega\,\mathrm{d}\omega}{1-(\frac{1}{8}al^2\cos^2\omega)^2} \tag{6}$$

According to equation (4) we must have $\frac{1}{8}al^2\cos^2\omega<1$. Thus, we may expand $1/(1-(\frac{1}{8}al^2\cos^2\omega)^3)^{1/2}$ in a binomial series:

$$1/[1-(\tfrac{1}{8}al^2\cos^2\omega)^2]^{1/2} = \sum_{n=0}^{\infty}\frac{(2n)!}{2^{2n}(n!)^2}(\tfrac{1}{8}al^2)^{2n}\cos^4\omega$$

Substituting it into equation (6) and integrating, we have

$$y = -\sum_{n=0}^{\infty}\frac{(2n)!}{2^{2n}(n!)^2}(\tfrac{1}{8}al^2)^{2n+1}\times$$

$$X\sum_{r=0}^{2n+1}\frac{2^{2n-2r+2}(2n+1)!^2(2r)!}{(4n+3)!(r!)^2}\cdot\left(1-4\frac{X^2}{l^2}\right)^r + D \tag{7}$$

According to the boundary condition $y(0)=0$, we have $D=0$. Substituting $x=\frac{1}{2}$ into equation (7), we have the deflection at the middle point of span:

$$y_{1/2} = -\sum_{n=0}^{\infty}\frac{2(2n)!(2n+1)!^2}{(n!)^2(4n+3)!}(\tfrac{1}{8}al^2)^{2n+1}\cdot l \tag{8}$$

where y is measured as a positive upward value. By taking the first term, equation (8) is the same equation as equation (1).

Influence of shearing action on deflection

We have derived the equation for calculating beam deflection without taking shear force into consideration. If this effect is being considered, the slope of the deflection curve due to shear force approximately equals the shear strain at the neutral axis. Thus, we have

$$\begin{cases} \mathrm{d}y_s/\mathrm{d}x = v_s \\ v_s = \dfrac{1}{G}\alpha_s\dfrac{Q}{A} \end{cases} \tag{9}$$

where G is the shear modulus, α_s is shearing coefficient, Q is shear force on the section, y_s is the deflection due to shearing action and v_s is shear strain of neutral axis, respectively.

Solving equation (9) we have

$$y_s = \frac{1}{G}\alpha_s\frac{Q}{Ax} + C' \tag{10}$$

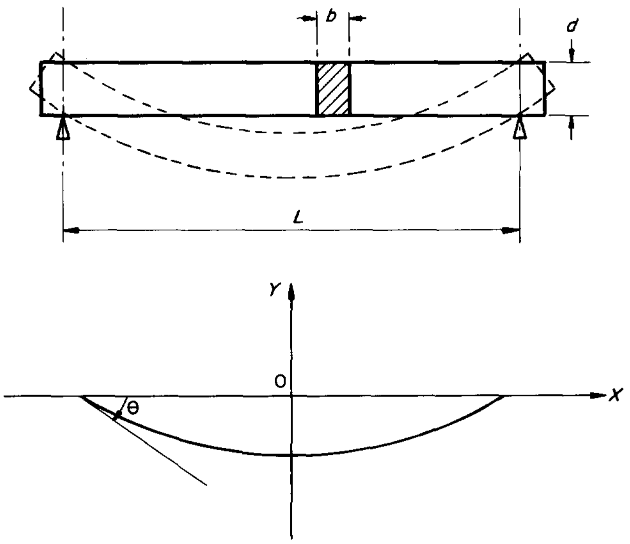

Figure 3 Representation of a simple supported beam with concentrated load

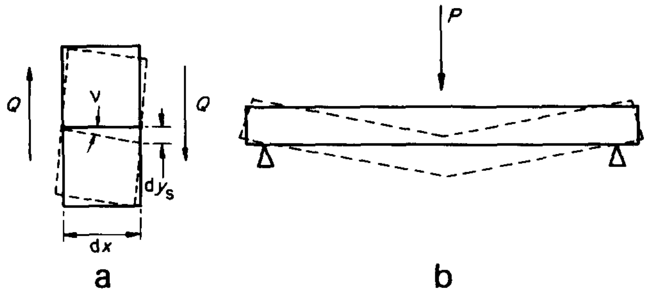

Figure 4 Diagram of deflection due to shearing action

According to boundary condition $y_s(0)=0$, we have $C'=0$. Then, we have

$$y_s = \frac{1}{G}\alpha_s \frac{Q}{A} x \tag{11}$$

From the elasticity theory viewpoint

$$\alpha_s = \frac{12+11\mu}{10(1+\mu)} \quad \text{and} \quad G = \frac{E}{2(1+\mu)}$$

where μ is the Poisson ratio of the material.

Substituting the above relations into equation (11) we obtain

$$y_s = \frac{(12+11\mu)}{10Ebd} PX \tag{12}$$

At the middle portion of the beam, the deflection due to shear force is

$$y_{s \cdot 1/2} = \frac{(12+11\mu)}{20E\,bd} PL \tag{13}$$

Thus, the deflection at the middle point of span, taking the shear force into consideration, may be written as

$$y_{1/2} = \sum_{n=0}^{\infty} \frac{2(2n)!(2n+1)!^2}{(n!)^2(4n+3)!}\left(\tfrac{1}{8}al^2\right)^{2n+1} \cdot L + \frac{(12+11\mu)}{20Ebd} PL \tag{14}$$

Similarly we can write the calculating equation of small deformation, taking into consideration the shearing action, as follows:

$$y_{1/2} = \frac{1}{48} \cdot \frac{PL^3}{EI} + \frac{(12+11\mu)}{20E\,bd} PL \tag{15}$$

On elastic mechanics solution for deflection curve equation

Let ϕ represent the stress function, which may be subjected to following compatibility condition

$$\nabla^4 \phi = 0 \tag{16}$$

The stress–stress function relation is:

$$\left.\begin{aligned} \sigma_x &= \frac{\partial^2 \phi}{\partial y^2} \\ \sigma_y &= \frac{\partial^2 \phi}{\partial x^2} \\ \tau_{xy} &= -\frac{\partial^2 \phi}{\partial x \partial y} \end{aligned}\right\} \tag{17}$$

where σ_x is the normal stress in the X direction, σ_y is that in Y direction and τ_{xy} is shearing stress in the Y direction with an X plane (or the shearing stress in the X direction with a Y plane), respectively. Suppose that at the boundary $y=0$, a local uniformly distributed load is q. Boundary conditions

$$\sigma_y\Big|_{y=0} = -q(z), \qquad \sigma_x\Big|_{x=0} = 0, \qquad \tau_{xy}\Big|_{y=0} = 0,$$

$$\sigma_y\Big|_{y=d} = 0, \qquad \sigma_x\Big|_{x=1} = 0, \qquad \tau_{xy}\Big|_{y=d} = 0,$$

$$\int_0^d \tau_{xy}\Big|_{x=0} \mathrm{d}y = R, \qquad \int_0^d \tau_{xy}\Big|_{x=1} \mathrm{d}y = -R$$

where R is the reaction at the supports for a simple beam and $q(x)$ is a piecewise continuous function

$$q(x) = \begin{cases} 0 & 0 \leqslant x < \frac{1}{2}l - \delta \\ q & \frac{1}{2}l - \delta \leqslant x < \frac{1}{2}l + \delta \\ 0 & \frac{1}{2}l + \delta \leqslant x < L \end{cases}$$

Then we may choose the stress function ϕ as follows[7]:

$$\phi = \sum_{n=1}^{\infty} \left(A'n\,Sh\frac{n\pi y}{L} + B'n\,Ch\frac{n\pi y}{L} + C'n\,y\,Sh\frac{n\pi y}{L} + D'n\,y\,Ch\frac{n\pi y}{L} \right) \sin\frac{n\pi x}{L}$$

where $A'n$, $B'n$, $C'n$ and $D'n$ are indeterminate constants. The stress–strain relation (generalized Hooke's Law) is:

$$\left.\begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \mu\sigma_y) \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \mu\sigma_x) \\ v_{xy} &= \frac{2(1+\mu)}{E}\tau_{xy} \end{aligned}\right\} \tag{18}$$

where ε_x is the strain in the x direction, ε_y is that in the y direction and v_{xy} the shearing strain, respectively.

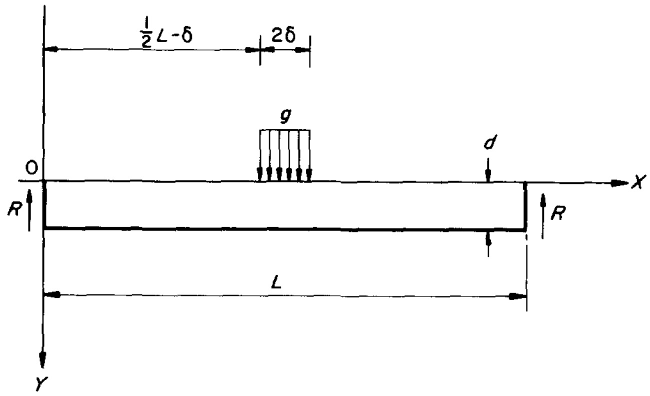

Figure 5 Diagram for elastic mechanics method

The strain–displacement relation (geometric equation) is:

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ v_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{aligned}\right\} \tag{19}$$

where u is the displacement in the x direction and v is that in the y direction.

The boundary conditions are:

$$u\Big|_{\substack{x=0\\y=d}} = 0 \quad \text{and} \quad v\Big|_{\substack{x=1\\y=d}} = 0$$

From the equations (17), (18) and (19) we find the solution of displacement in the y direction is:

$$V = -\frac{2Pl}{Eb}\sum_{n=1,3,5\ldots}^{\infty}\frac{n\pi}{l}\left\{2An\,Ch\frac{n\pi d}{l} + \left[(1+\mu)Bn - \frac{l}{n\pi}Cn\right]Sh\frac{n\pi d}{l} + (1+\mu)d\,Cn\,Ch\frac{n\pi d}{l} + (1+\mu)d\,Dn\,Sh\frac{n\pi d}{l}\right\} \tag{20}$$

where

$$An = -\frac{1}{n^3\pi^3}\cdot\frac{d + \frac{L}{n\pi}Sh\frac{n\pi d}{L}Ch\frac{n\pi d}{L}}{d^2 - \frac{L^2}{n^2\pi^2}Sh^2\frac{n\pi d}{L}}, \qquad Bn = -\frac{1}{n^2\pi^2}$$

$$Cn = -\frac{L}{n^3\pi^3}\cdot\frac{Sh^2\frac{n\pi d}{L}}{d^2 - \frac{L^2}{n^2\pi^2}Sh^2\frac{n\pi d}{L}}$$

$$D_n = -\frac{1}{n^2\pi^2}\cdot\frac{d + \frac{1}{n\pi}Sh\frac{n\pi d}{L}Ch\frac{n\pi d}{L}}{d^2 - \frac{L^2}{n^2\pi^2}Sh^2\frac{n\pi d}{L}}$$

In solving the above expressions we assume $\delta \to 0$ and $2q\delta \to P$. This treatment implies the above case to be commensurate with the case of a concentrated load. If the above notation is adopted, we may write the deflection expression, at the middle point of span, as follows:

$$y = \frac{2PL}{Eb}\sum_{1,3,5\ldots}^{\infty}\frac{n\pi}{L}\left\{-2\,An\,Ch\frac{n\pi d}{L} - \left[(1+\mu)Bn - \frac{L}{n\pi}Cn\right]Sh\frac{n\pi d}{L} - (1+\mu)d\,Cn\,Ch\frac{n\pi d}{L} - (1+\mu)\,d\,Dn\,Sh\frac{n\pi d}{L}\right\} \tag{21}$$

where An, Bn, Cn and Dn are the constants as mentioned in equation (20).

EXPERIMENTAL

Preparation and procedure

Commercial poly(vinyl chloride) (PVC) and styrene–butadiene rubber (SBR) were the materials used in this study. The PVC and SBR properties are listed in *Table 1*. SBR/PVC blend was prepared by means of mill mixing in a two-cylindrical roll. The blending test was carried out according to ASTM D790[8] in a universal testing machine made by the Changchun testing machine plant. A SBR/PVC blend sheet was cut into beams 10 (mm) wide, 15 (mm) thick and 150 (mm) long for test specimens. Using a mechanical strain gauge, bending deflection data were measured directely. The data measured before onset of yielding may be taken as those in the elastic range.

The value of Young's modulus and Poisson's ratio were measured in same testing machine, with the aid of a modified extensometer.

Data and data analysis

The content (weight ratio) ranges are presented in *Table 2*. For a number of SBR/PVC systems, we measured deflection data under different loads. We have plotted the devlection values *versus* load in the case of weight ratio SBR/PVC = 7/93. The results are shown in *Figures 6–9*. In order to make the results clearer, we have depicted them as so-called relative deviation graphs as shown in *Figures 10–15*. The full circles are the experimental data and the full lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively. In this paper we define the relative deviation as follows:

$$\frac{y_e - y_t}{y_t}100\ (\%)$$

where y_e is experimental value of deflection and y_t is theoretical value of deflection.

In view of all the results given above, it will be seen that the differences between the experimental data and theoretical values calculated from equation (1) are clear, especially where the Young's modulus of the blend is small (e.g. when SBR/PVC ratio = 30/70, relative deviations exceed 5%). As the load increases the error also increases gradually. The theoretical values of deflection calculated from equation (8) are in good quantitative agreement with the measurements. For various cases the relative deviations are less than 2%. In

Table 1 PVC and SBR properties

Sample	M_w	T_g
PVC	54 000	85°C
SBR	276 000	−59°C

Table 2 Content of SBR/PVC blend

Sample	SBR (%)	PVC (%)
1	0	100
2	7	93
3	10	90
4	15	85
5	20	80
6	30	70

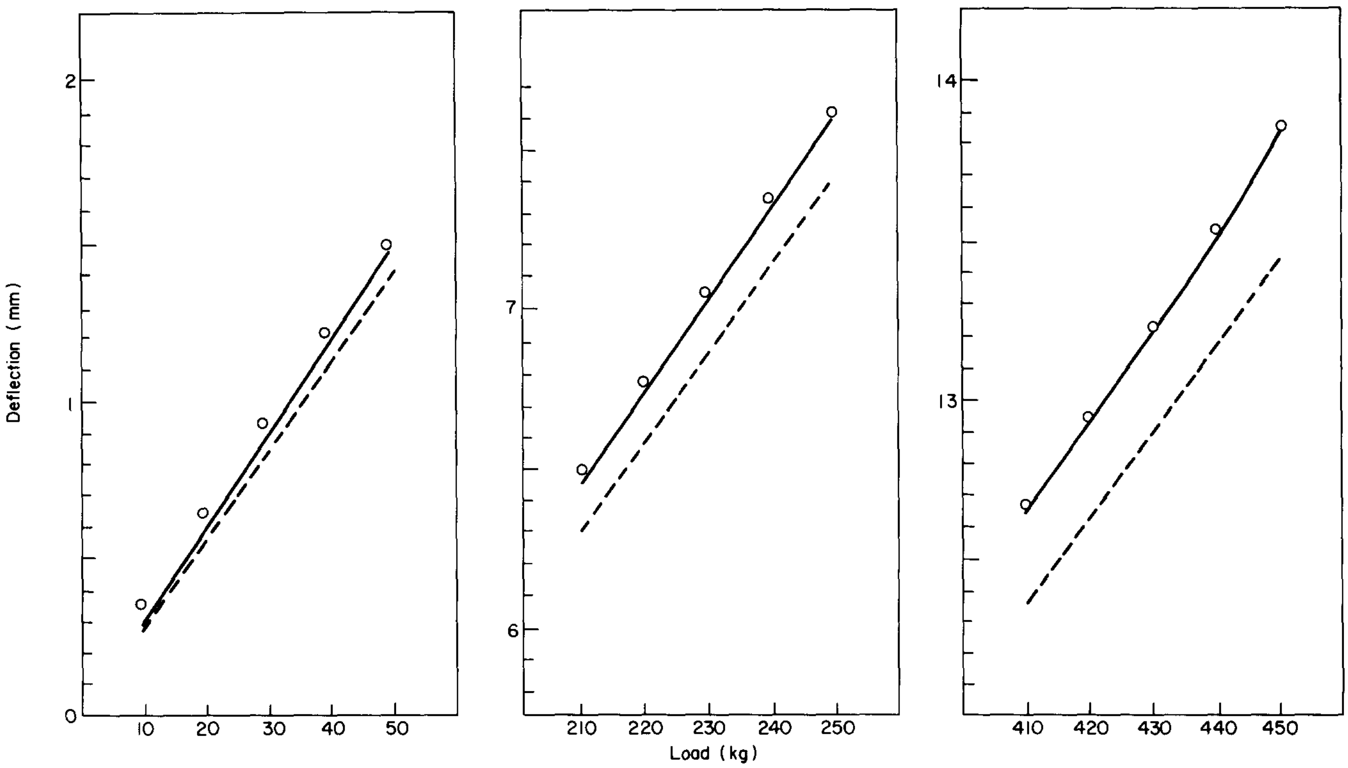

Figure 6 Deflection of SBR/PVC blend (7/93) *versus* load; (○), experimental data; solid line, theoretical values calculated from equation (8); broken line, theoretical values calculated from equation (1)

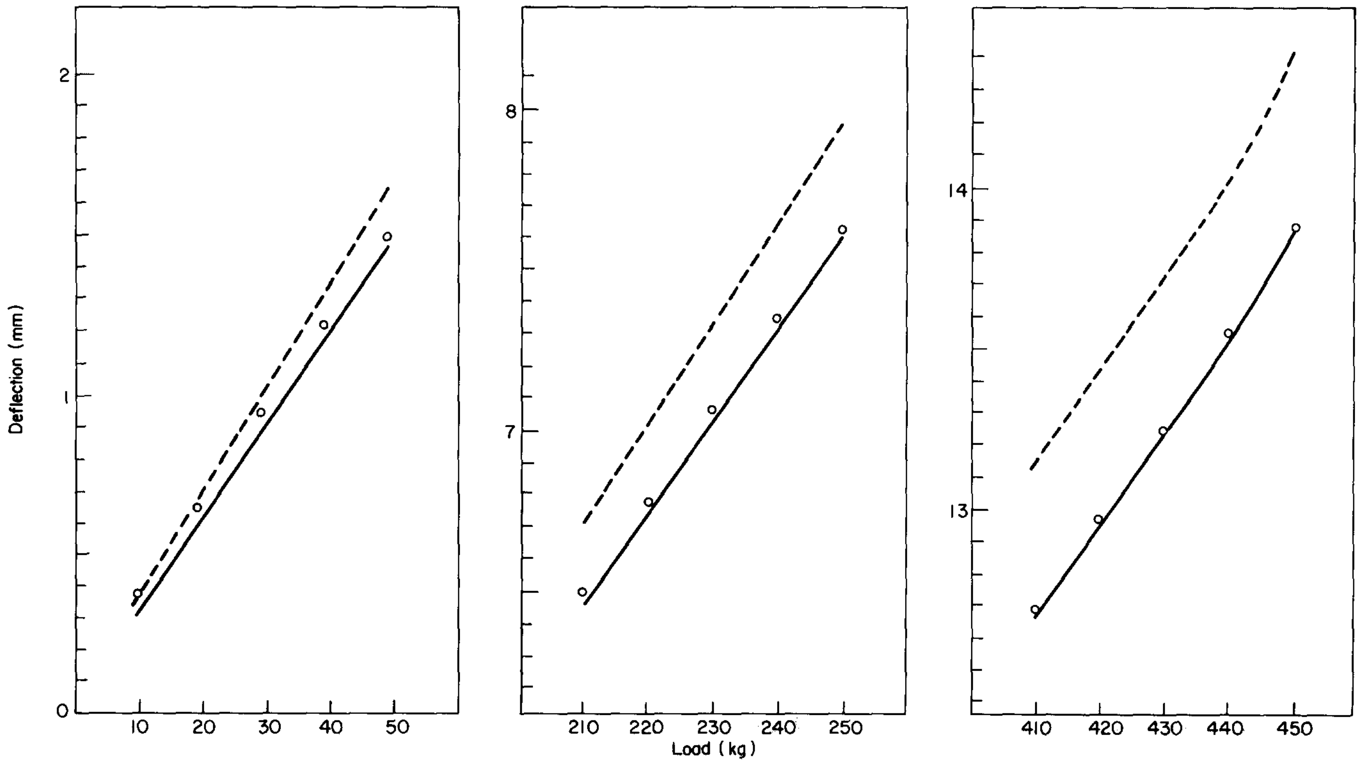

Figure 7 Deflection of SBR/PVC blend (7/93) *versus* load; (○), experimental data; solid line, theoretical values calculated from equation (8); broken line, theoretical values calculated from equation (14)

case of large deformation equation (8) is more accurate than the others. Equations (14) and (15) are unsuitable for predicting deflection of SBR/PVC blends. The values of deflection calculated from equations (14) and (15) are clearly on the high side. The deflection values calculated from the elastic mechanics method (equation (21)) appear to approach the measured values. For the most part the relative deviation might range from 1 to 3%. Its relative deviation is less than the others in the case of small deformation.

CONCLUSION

In summary it may be concluded that equations (1), (15) and (21) are all linear, as they all assume a small

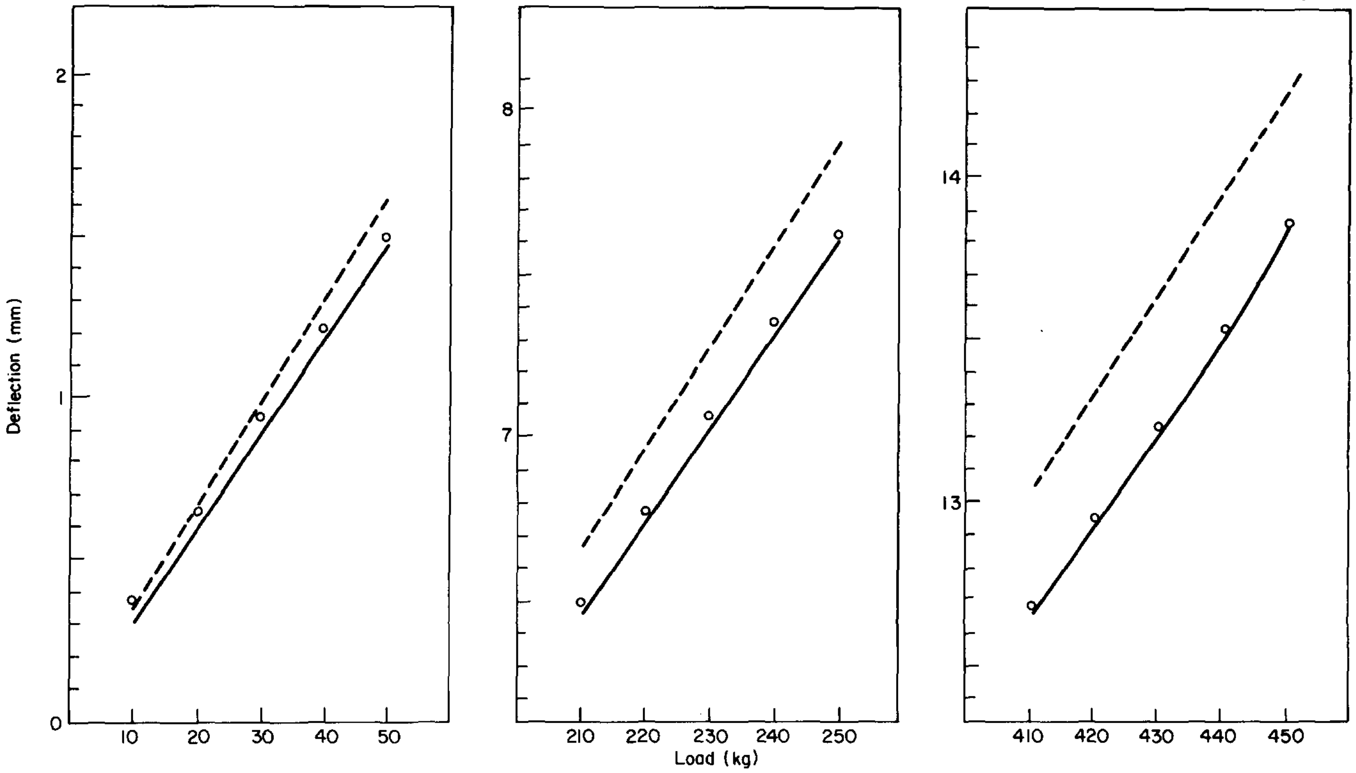

Figure 8 Deflection of SBR/PVC blend (7/93) *versus* load: (○), experimental data; solid line, theoretical values calculated from equation (8); broken line, theoretical values calculated from equation (15)

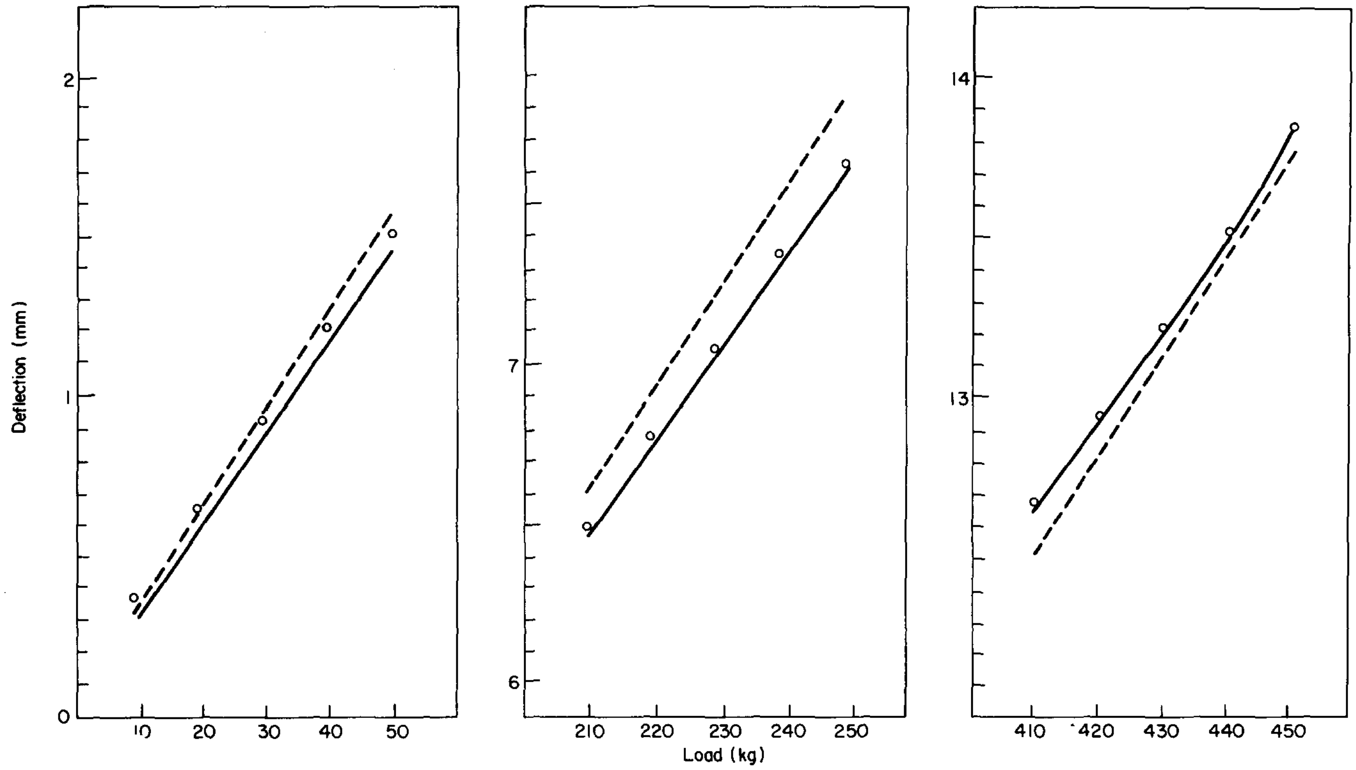

Figure 9 Deflection of SBR/PVC blend (7/93) *versus* load: (○), experimental data; solid line, theoretical values calculated from equation (8); broken line, theoretical values calculated from equation (21)

deflection*. However, we have been concerned with a number of different factors in these three equations. Equation (1) only relates to the effect of bending action in the case of small deflections. Equation (15) deals not only with the deflection due to bending action but also that due to shearing action. In equation (21) the deflection is given by solution of the elastic mechanics. It refers comprehensively to the effects of bending and shearing action, according to the basic equations (balance, physical and geometric equations), compatibility conditions and boundary conditions. Therefore, these

* i.e. let $y''[1+(y')^2]^{-3/2}=y''$. This is only a geometric assumption and not a physical concept.

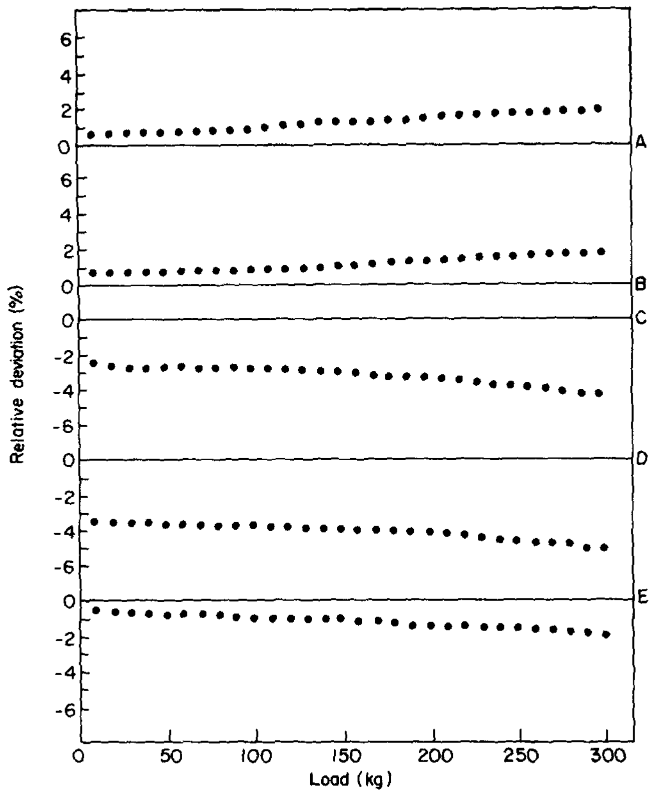

Figure 10 Relative deviation graph of SBR/PVC blend (0/100): (●), experimental data; solid lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

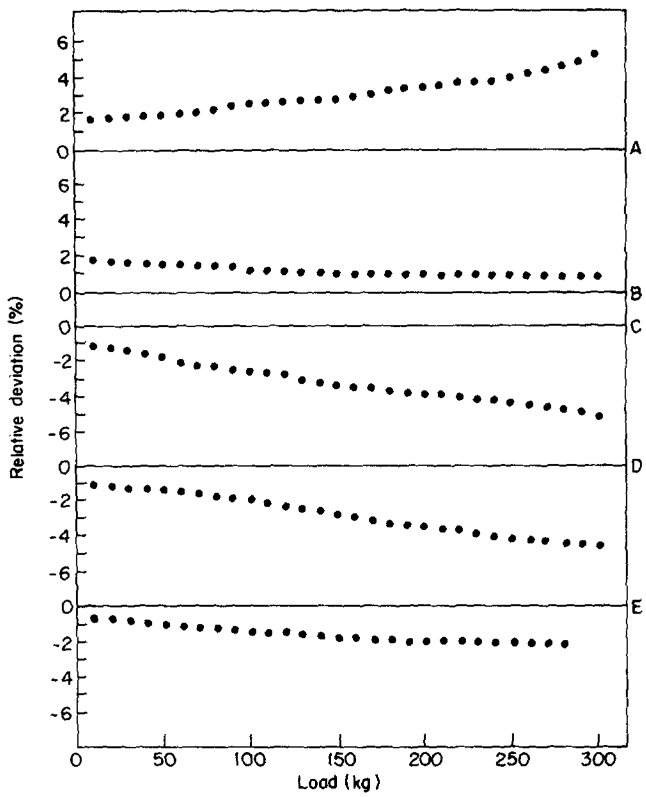

Figure 12 Relative deviation distribution graph of SBR/PVC blend (10/90): (●), experimental data; full lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

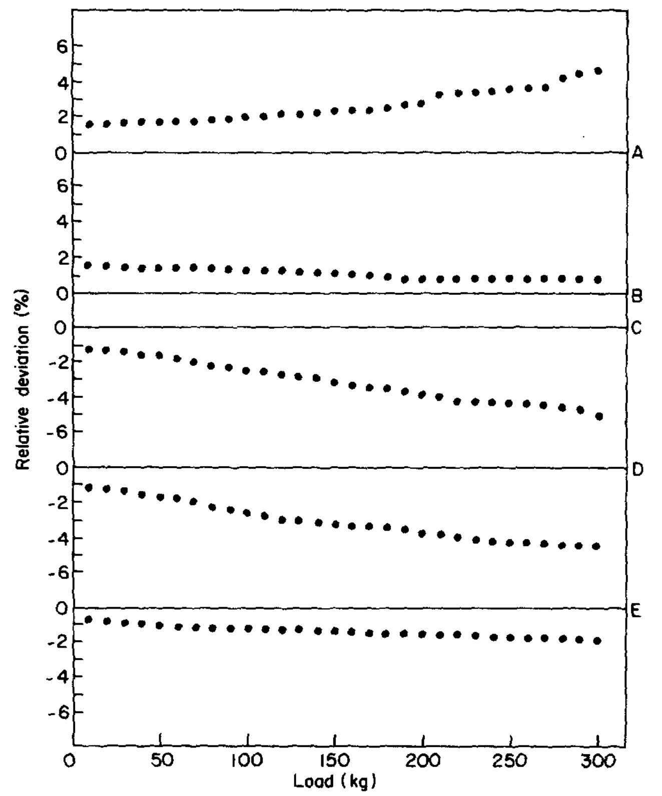

Figure 11 Relative deviation distribution graph of SBR/PVC blend (7/93): (●), experimental data; solid lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

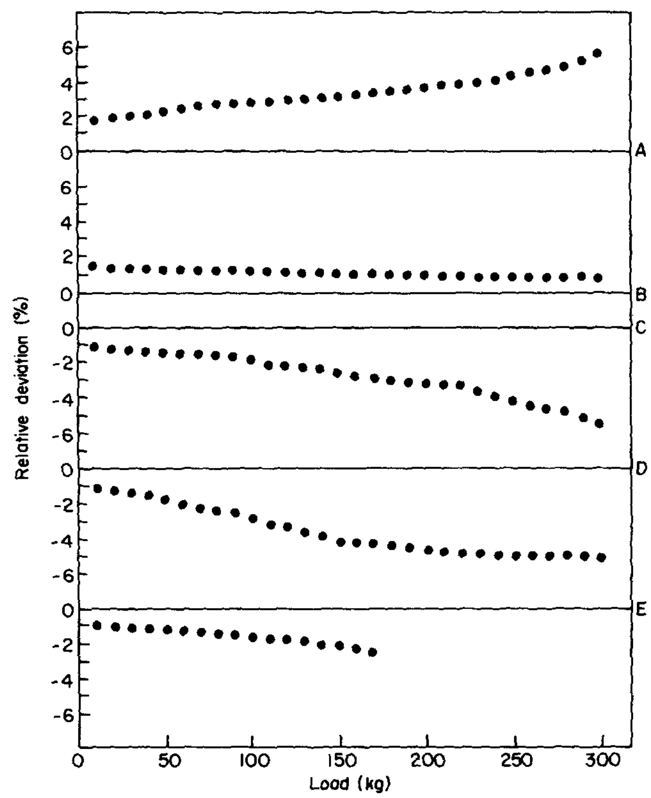

Figure 13 Relative deviation distribution graph of SBR/PVC blend (15/85): (●), experimental data; solid lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

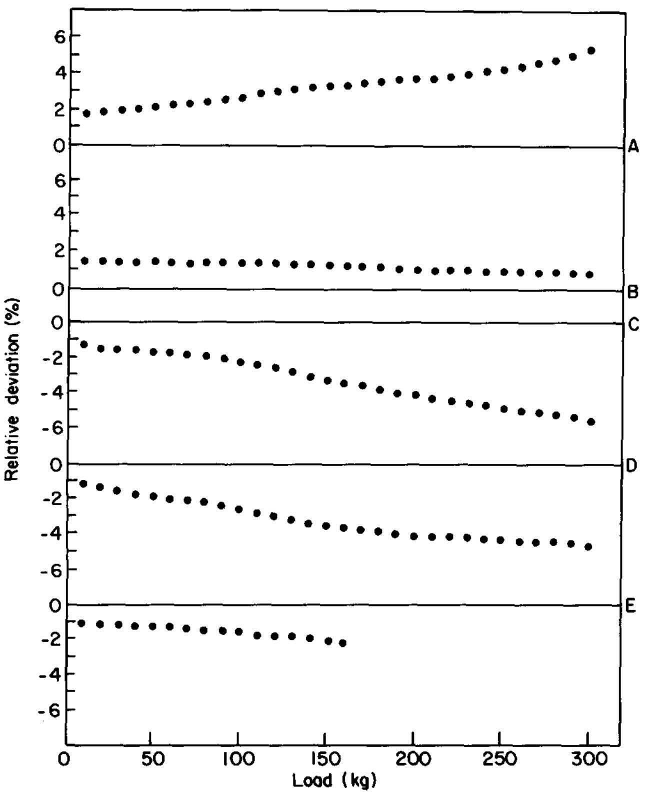

Figure 14 Relative deviation distribution graph of SBR/PVC blend (20/80): (●), experimental data; solid lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

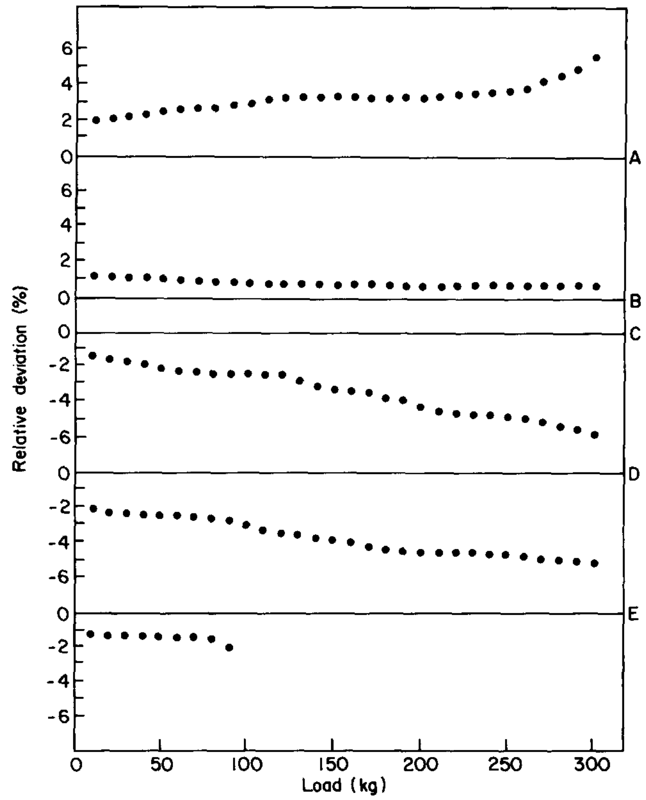

Figure 15 Relative deviation distribution graph of SBR/PVC blend (30/70): (●), experimental data; solid lines A, B, C, D and E are theoretical values calculated from equations (1), (8), (14), (15) and (21), respectively

three equations indicate how departures from experiment can vary with the applied load. Checking the theoretical value against the experimental value, we found that the predicted value calculated from equation (21) is more successful than the others in the case of small deflections. This is because equation (21) is concerned simultaneously with the influences of bending and shearing actions. Although equation (15) has also a shearing term, its value has a tendency to be on the high side and it may be explained by following reasons: the shear force at any section on the left and the right of the beam are numerically equal, but opposite in direction, thus, the warping due to shear force at any section on the left and the right of the beam are also numerically equal and opposite in direction. It is known that the section located at the middle point of the beam must be kept in a plane without any warp, as geometric symmetry of the beam, that is to say, the warping of this section was constrained. This is the very warping constraints that prevented bending deformation of the beam. Therefore, the practical values must be less than those calculated from equation (15). The error in equation (14) can be explained by the same reason. Equation (8) is a non-linear equation also. The predicted value calculated from equation (21) is better than that from equation (8) in the case of small deflections and by implication, the influence due to non-linearity is less than that due to shearing action in this case. The results show that the predicted value calculated from equation (8) is more accurate than for the other equations in case of large deflections. As mentioned above, equations (1), (15) and (21) are all linear, and it seems that the influence due to non-linearity is greater than that due to shearing action in this case. For the SBR/PVC blend, we recommend using equation (8) to calculate deflection.

Finally, let us emphasize again that above results hold good in the range of elasticity. More detailed studies including other fields are in progress.

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